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ON THE RAPID CALCULATION OF TIMES OF MOONRISE AND MOONSET.

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On the rapid calculation of times of moon-rise and moon-set. *By* J. H. FIELD, M.A., *Imperial Meteorologist, and* S. M. JACOB, I.C.S.

There has arisen of late years in India a considerable demand on the Meteorological Department for information regarding the times of rise and set of sun and moon. During a large part of each year the weather conditions in this country are sufficiently definite to allow of the construction of street lighting tables, based on sunlight and moonlight, and it has accordingly come to be realised in large towns that a great saving of oil or other illuminant can be arranged for when precise information is available as to the hours of darkness to be expected.

2. As long as it was required to make the calculations for one place only, the routine method described in paragraphs 3 to 6 was adopted.* When, however, the necessity arose of computing times of set and rise of the moon for many places throughout each year, the work involved became sufficiently great to call for shortened methods; and the principal object of this note is to show (paragraphs 7 to 11) how, when data for the moon have once been calculated for a station A by an unskilled computer, the calculation can be subsequently extended to any other geographical positions B, C, etc., with comparatively little further expenditure of time.

3. For the routine determinations for station A it is first necessary to calculate a table of hour-angles in terms of declination (*i. e.,* of time intervals from meridian passage for all declinations), and then to apply the particular values of hour-angle, relating to the declination at each rise and set, to the daily times of meridian transit, separately calculated.

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* This preliminary routine method, paragraphs 3 to 6, probably contains nothing intrinsically novel, but has been described because it has proved satisfactory where only unskilled computation is available, subject merely to scrutiny of results for consistency, and to examination of the working for a day here and there.

The hour-angle declination table is obtained as follows :—

In fig. 1 P is the pole, z the zenith, Q Q the equator, h h the horizon, m the moon.

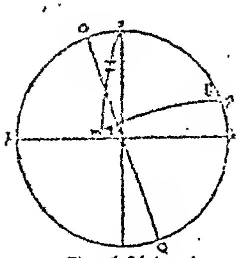

Fig. 1: Sketch 1

Then Ph=latitude ϕ.

δ =declination, + when north.

Z=zenith distance.

θ=hour-angle.

Pm=90—δ.

Pz=90—ϕ.

Therefore in the triangle Pm z, $\cos Z = \sin\delta \sin\phi + \cos\delta \cos\phi \cos\theta$

$$\therefore \cos\theta = \frac{\cos Z - \sin\delta \sin\phi}{\cos\delta \cos\phi} \quad \ldots\ldots\ldots\ldots (1.)$$

The calculation of θ for a given place results, for the latitude of Allahabad, in a curve such as fig. 2: Z has there been taken as 90° minus 45′ to allow for refraction, parallax, and for a position of the moon such that its lower limb just touches the horizon.* This is the most convenient position to consider, since it is then unnecessary to go into the question of the orientation of the illuminated sector, which varies with the moon's age.

4. With the hour-angle table ready, it now only remains to calculate the daily times of meridian passage in local mean time, apply the hour angle proper to the declination for the time of set or rise, and then transform into civil time for the standard meridian of the country.

Now the L. M.† time of meridian passage at any place differs from the G. M. T. of Greenwich meridian passage merely by the amount of change of moon's R. A. in the time interval between the two meridians. The daily figures for Greenwich are given in N. A. (table IV for each month) and from this daily difference, due to daily change in R. A., there follows the amount of the correction required.

It is necessary next to find from the curve, fig. 2, the hour-angles to be applied to the time of meridian passage just found, and to do this the moon's declination at rise and set must be known. The declination is given in the N. A. for each hour of each day (tables V to XII monthly) but appears there in G. M. A. T. It is necessary therefore to determine approximately the times of local rise and set, put these into G. M. A. T. and then enter the declination from the almanac. For the present purposes the times need only be approximate since declination changes slowly, and they may be taken as 6 hours before and after meridian passage.‡

* The angle 90°—45′ is not exactly the zenith distance with these conditions, but is sufficiently near for practical purposes; while its adoption has the distinct advantage that a curve drawn for it, fig. 2, but with its scales of time and declination reversed in direction serves also, to the same approximation, for the sun with zenith distance 90°+45′, i.e., for the first ray at rise and last at set, refraction and angular diameter being taken into account. (The actual zenith distances in these two cases should be 90°—40′ for the moon, and 90°+49′ for the sun.)

† L. M. = local mean, G. M. T. = Greenwich mean time, R. A. = right ascension, N. A. = Nautical Almanac, G. M. A. T. = Greenwich mean astronomical time, L. M. A. T. = local mean astronomical time.

‡ When the moon is off the equator the final error introduced by assuming for the declination that rise and set are 6 hours from meridian passage is never greater for India than 1 minute of time, but would be increased with latitudes above that of Peshawar (34°), which represents the extreme north of India.

The sums of the time of meridian passage and the hour-angles have finally to be corrected for change of the moon's R. A. in the latter intervals, and this is done by reference to the N. A. for rate of change at the time.

5. The actual process of computation is most conveniently done by means of the tabular statement below, in which the columns are to be filled in by the computer with the figures indicated in the explanatory notes below it.

	Col. 1	2	3	4	5	6	7	8	9	10	11	12	13	14
(Set) ..	9·4·11 etc.	9·27	95	10·8	9·16·2	9·46	14 N.	6·23·5	+13	6·36·5	15·52·7	27·55·2	3·55·2	10th
(Rise) ...	9th	"	"	"	"	21·46	17N.	6·30	+14	6·44	2·3 2·2	14·34·7	...	9th

The example given is for Allahabad in long. 81° 54′ E. lat., 25° 28′ N; and the following notes apply :—

Column 1; date.

" 2; Greenwich Mean Astron. Time of Greenwich upper meridian passage, to nearest minute, direct from table IV monthly in nautical almanac.

" 3; Change of column 2 in 48 hours, to nearest minute.

" 4; Col. 3 × n/48 where n = hours east of Greenwich: (·114 for Allahabad).

" 5; Col. 2—col. 4; local mean astron. time of local meridian passage.

" 6; G. M. A. T. of local moon set or rise (sufficiently accurate if put equal to col. 5 in G. M. A. T. + and — 6 hours; + for set and — for rise).

" 7; Moon's declination (nearest ½°) at time of col. 6.

" 8; Hour-angle (to ½ min.), from curve of θ, δ; plus sign for set, minus sign for rise.

" 9; Correction (to ½ min.), for change in R. A. of moon during time col. 8 round about time of col. 5. This latter should really be put into G. M. A. T., but as R. A. changes slowly, the error in using col. 5 is not more than ½ min. for any longitude. Signs as in col. 8.

" 10; Col. 8 + Col. 9; lapse of time since meridian passage.

" 11; Col. 5 + Col. 10; L. M A. T. of local rise or set.

" 12; Col. 11 + 12 hours, to reduce to Civil Time, + correction (for Allahabad = + 2½ minutes) to the nearest standard meridian, 82½°E. for India.

" 13; } correction of date, *i.e.*, 27 h. 55·2 m. on 9th becomes 3 h. 55·2 m. on 10th.
" 14; }

6. With the approximations employed in the tabular form, the error in the various steps will not exceed, for India,

Column 2	...	...	$\pm$	0·5	minute.
Columns 3 and 4	...	...	$\pm$	0·1	"
Column 6	...	...	$\pm$	1·0	"
" 7	...	...	$\pm$	0·7	"
" 8	...	...	$\pm$	0·25	"
" 9	...	...	$\pm$	0·25	"

or the maximum possible accumulated error will be $\pm$ 2·8 min. in the times of rise and set.

Method of extending the results of Station A to a number of other geographical positions.

7. The additional process now to be described, is used for all further places requiring moonlight information, and is based on the fact that it is easier to transform times from a station, A, preferably on the equator* to a series of other geographical positions B, C, etc., than to calculate each of the latter separately by the process already described.

With the notation employed in paragraph 3 we have :—

$$\cos \theta = \frac{\cos Z - \sin \phi \sin \delta}{\cos \theta \cos \delta} = a, \text{ say}$$

so that

$$\theta_1 - \theta_2 = \cos^{-1} a - \cos^{-1} b = \cos^{-1} k, \text{ say}$$

$$\therefore \quad k = \cos(\cos^{-1} a - \cos^{-1} b)$$

$$= ab + \sin(\cos^{-1} a) \sin(\cos^{-1} b)$$

$$= ab + \sqrt{(1 - a^2)(1 - b^2)}$$

$$\therefore \quad \theta_1 - \theta_2 = \cos^{-1}\left[ab + \sqrt{(1 - a^2)(1 - b^2)}\right]$$

When $Z = 90°$, as in the case of a star, with refraction left out of account, we have :—

$$\theta_1 - \theta_2 = \cos^{-1}\left[\tan \phi_1 \tan \phi_2 \tan^2 \delta + \sqrt{(1 - \tan^2 \phi_1 \tan^2 \delta)(1 - \tan^2 \phi_2 \tan^2 \delta)}\right]$$

put $\phi_1 = 0$, *i.e.* assume a place on the equator,

then

$$\theta_1 - \theta_2 = \cos^{-1} \sqrt{1 - \tan^2 \phi_2 \tan^2 \delta}$$

$$= \sin^{-1}(\tan \phi_2 \tan \delta).$$

$$\therefore \quad \sin(\theta_1 - \theta_2) = \tan \phi_2 \tan \delta$$

a result calling for the application of a small correction, paragraph 10, when Z is not made exactly 90°.

8. This equation has been worked out for various latitudes, each in steps of 5° of declination, giving $\theta_1 - \theta_2$ in intervals of sidereal time. After correction to mean-time intervals, and allowing for the change in R. A. of the moon during those intervals (at an average rate of 2·2 secs. in 1 min., an average which gives a maximum error of 1 min. for latitudes less than 35°), the following figures for $\theta_1 - \theta_2$ are obtained :—

---

* The use of an equatorial station for the original calculation has the advantage that to the required approximation the declination has no influence on the hour angle, which for the moon remains constant at 5 hours 57 minutes, and for the sun constant at 6 hours 3 minutes, rise and set being defined as in paragraph 3 and its footnote.

This constancy introduces a simplification in the calculations of paragraph 5, since the variable of column 8 becomes a constant. For the sun the simplification is still greater, and the calculation of rise and set in L. M. T. for any position merely consists in applying the equation of time and the corrections for latitude (from figure 3) to the constant hour-angle 6 hours 3 minutes.

δ	$\phi_1=10°$		15°		20°		25°		30°		35°	
	Min.	Sec.	Min.	Sec.	Min.	Sec.	Min.	Sec.	Min.	Sec.	Min.	Sec.
5	3	39	5	35	7	34	9	39	12	16	14	33
10	7	22	11	10	15	14	19	30	24	11	29	21
15	11	09	17	02	23	09	29	41	36	48	44	44
20	15	14	23	09	31	30	40	25	50	11	61	04
25	19	30	29	41	40	25	51	58	64	34	78	51
30	24	10	36	48	50	11	64	34	80	29	98	37

These are shown plotted in fig. 3.

9. The transformation is most conveniently made when an equatorial position has been chosen for the place of reference A, but if for any reason the original figures relate to another latitude (*e.g.*, Allahabad in the example above) the transformation can obviously be effected by employing the *difference* between the corrections, from fig. 3, towards the equator and away from it to the new situations under treatment. An illustration of the application of the transformation table may be useful. Taking Allahabad, lat. 25° 28′N long. 81° 54′E, as the station of reference, and starting with col. 12 as previously determined in paragraph 5, the times for Peshawar, lat. 34° 1′N, long. 71° 35′E, may be obtained as shown in the following table:—

	Col. 7	*8*	*9*	*10*	*11*	*12*	A	B	C	D
set 9th	*14°N*					*27-55·2*	+11·5	28-49·7	4-49·7	10th
rise 9th	*17°N*					*14-34·7*	—14	15-3·7	...	9th

Here the columns 7 and 12 in italics are those already worked out, *in extenso* for Allahabad, and the blank table for the new station is merely to be clipped to the table of figures for Allahabad while the remaining work, columns A to D, is done. Column A contains the latitude corrections from the curves, figure 3, for the declinations of column 7, and in the present case is made up of the difference of correction from Allahabad towards the equator, and away from it to Peshawar.

Thus in set, declination N, we have $-28{\cdot}5+40=+11{\cdot}5$ min.

„ rise, „ N, „ $+35-49=-14$ min.

(for declination S, the signs would be reversed.)

6. With the approximations employed in the tabular form, the error in the various steps will not exceed, for India,

Column 2	...	...	±	0·5	minute.
Columns 3 and 4	...	...	±	0·1	"
Column 6	...	...	±	1·0	"
" 7	...	...	±	0·7	"
" 8	...	...	±	0·25	"
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or the maximum possible accumulated error will be ± 2·8 min. in the times of rise and set.

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so that

$$\theta_1 - \theta_2 = \cos^{-1}a - \cos^{-1}b = \cos^{-1}k, \text{ say}$$

$$\therefore\quad k = \cos\,(\cos^{-1}a - \cos^{-1}b)$$

$$= ab + \sin\,(\cos^{-1}a)\,\sin\,(\cos^{-1}b)$$

$$= ab + \sqrt{(1-a^2)\,(1-b^2)}$$

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When $Z = 90°$, as in the case of a star, with refraction left out of account, we have :—

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This constancy introduces a simplification in the calculations of paragraph 5, since the variable of column 8 becomes a constant. For the sun the simplification is still greater, and the calculation of rise and set in L. M. T. for any position merely consists in applying the equation of time and the corrections for latitude (from figure 3) to the constant hour-angle 6 hours 3 minutes.

δ	$\phi_0 = 10°$		15°		20°		25°		30°		35°	
	Min.	Sec.	Min.	Sec.	Min.	Sec.	Min.	Sec.	Min.	Sec.	Min.	Sec.
5	3	39	5	35	7	34	9	39	12	16	14	33
10	7	22	11	10	15	14	19	30	24	11	29	21
15	11	09	17	02	23	09	29	41	36	48	44	44
20	15	14	23	09	31	30	40	25	50	11	61	04
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Thus in set, declination N, we have $-28{\cdot}5+40=+11{\cdot}5$ min.

" rise, " N, " $+35-49=-14$ min.

(for declination S, the signs would be reversed.)

Column B = column 12 + column A + a correction, constant throughout the year, composed as follows :—

(a) +41·5 min. the difference between the meridians at Peshawar and Allahābad.

(b) +1·5 min. the correction for change of the moon's R. A. during the interval (a).

Columns C and D allow, as before, for any change of date resulting from times in column B exceeding 24 hours, and only parts of them would be filled in.

10. *Errors.*—In the process of transformation as illustrated above, three small errors are added to those detailed for station A in paragraph 6.

We have first the error due to the assumption that over the range of latitudes considered, the obliquity of rise and set has a constant value, conveniently made equal to the mean value for the country in question. Second, the error arising from the change of declination of the moon during the interval $\theta_1-\theta_2$ for differences of latitude, and during the time interval involved by differences of longitude. Third, the error due to assuming in column B item (b) a uniform rate of change in R. A. of 2·2 secs. per min. of time.

To consider the first of these. The transformation curves have assumed $Z=90°$, and if, as is convenient, it is intended to give times for $Z=89°15'$, with the moon's lower limb just touching the horizon, a correction must be made, which varies to a slight extent with latitude, but for practical purposes can be taken as constant over a wide range of latitude.

If t be the minutes of time for the moon, with declination δ, to rise a minutes of arc in latitude ϕ we have as usual

$$t = \frac{a}{15\sqrt{\cos^2\delta - \sin^2\phi}}$$

and the greatest and least values of this, when $a=45'$, are 3·5 min. and 2·4 min. respectively, with the limits of latitude in India; and 3·3, 3·0 the corresponding values on the equator. If then we have calculated the hour-angles for station A with $Z = 90°-45'$, or if we apply once for all to the values calculated for $Z=90°$ a correction of 3 min. throughout in column 11, the neglected variation of this correction with latitude over the wide range required (say 34°N to 34°S), will introduce an additional error of only ±0·6 min. of time.

In regard to the second error, it is to be noted that for a given place the change of declination in the time interval $\theta_1-\theta_2$ is alternately plus and minus for both positive and negative declination, while during the smaller interval in longitude the change in declination changes from plus to minus once only in each complete swing in declination. The variation, with moon's declination, of this combined error is not therefore a simple curve, and it is best in cases where 1·5 min. is not of importance in the result to consider it an error, and not to attempt to correct it.

The third error never exceeds 0·8 min. for the whole range of longitude in India, when the equatorial place of reference has been chosen for a mean longitude.

The maximum accumulated error cannot then exceed $\pm$ (2·8+1·0+0·6+1·5+·8) $=\pm$ 6·7 min., and will in general be very much less.

11. With regard to the time taken for computation, it has been found that the complete year's figures of the station of reference A (on the equator or elsewhere), can be worked out and checked in 18 hours (3 days) of a clerk's time, and that the subsequent transformations for stations B, C, etc., in other geographical positions in India need take only 6 hours each per year. It is in general unnecessary to deal with the figures for rise and set which occur during day-time, or even within 1 hour of day-light, and in some case the periods covering the last and first quarters can be disregarded; the omission of these periods still further shortens the work.

Diagrams.

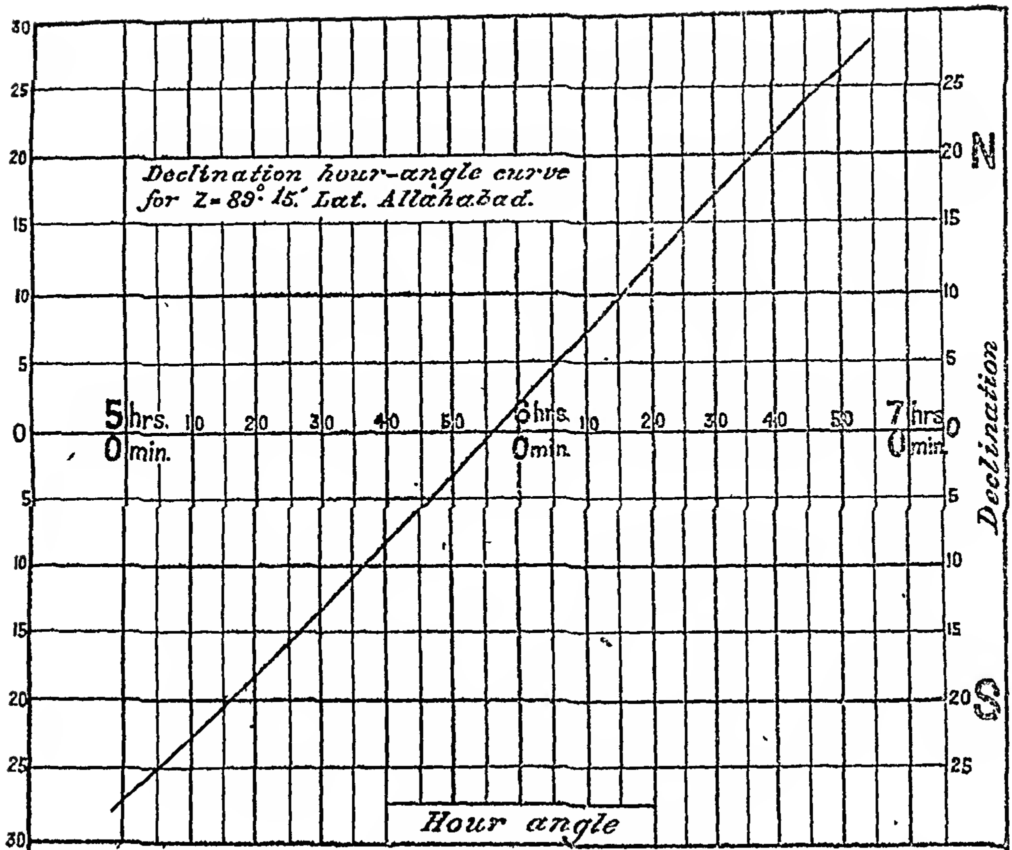
Figure 2 gives the hour-angle declination curve of the moon, when its lower limb just touches the horizon at rise and set, refraction and parallax being allowed for.

If the scales of time and declination are reversed in direction, the curve gives the hour-angle for the sun at first and last ray, refraction included.

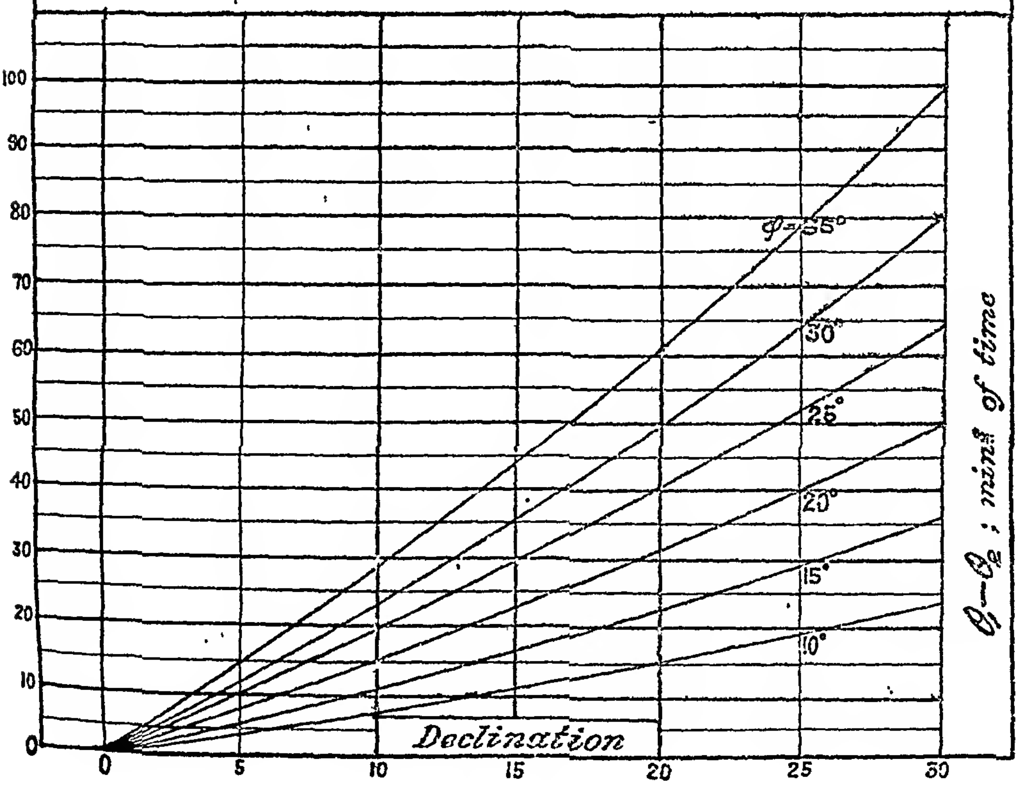
Figure 3 gives the correction necessary to obtain the hour-angles of rise and set in latitude ϕ, from the corresponding hour-angles on the same meridian at the equator. The correction is positive when hour-angle and declination are of similar sign, and negative otherwise.

G. C. B. P., Simla.—No. 174 M. R. G I.—14-10-12—400—D.W.

Fig. 2



Declination hour-angle curve for Z=89° 15.' Lat. Allahabad.
5 hrs. 0 min.
6 hrs. 0 min.
7 hrs. 0 min.
Hour angle
N
S
Declination



φ=35°
30°
25°
20°
15°
10°
Declination
θ1−θ2 ; mins of time

Fig. 3